Module-1 BJT AC Analysis:

BJT AC Analysis: BJT AC Analysis: BJT Transistor Modeling, The re transistor model, Common emitter fixed bias, Voltage divider bias, Emitter follower configuration. Darlington connection-DC bias; The Hybrid equivalent model, Approximate Hybrid Equivalent Circuit-Fixed bias, Voltage divider, Emitter follower configuration; Complete Hybrid equivalent model, Hybrid π Model.

BJT Transistor Modeling

- A model is an equivalent circuit that represents the AC characteristics of the transistor.
- Transistor small signal amplifiers can be considered linear for most application.
- A model is the best approximate of the actual behavior of a semiconductor device under specific operating conditions, including circuit elements

Transistor Models

- ✓ r_e- model – any region of operation, fails to account for output impedance, less accuracy
- ✓ Hybrid model – limited to a particular operating conditions, more accuracy

The r_e Transistor Model

BJTs are basically current-controlled devices; therefore the r_e models uses a diode and a current source to duplicate the behavior of the transistor. **One disadvantage to this model is its sensitivity to the DC level. This model is designed for specific circuit conditions.**

Common-Base Configuration

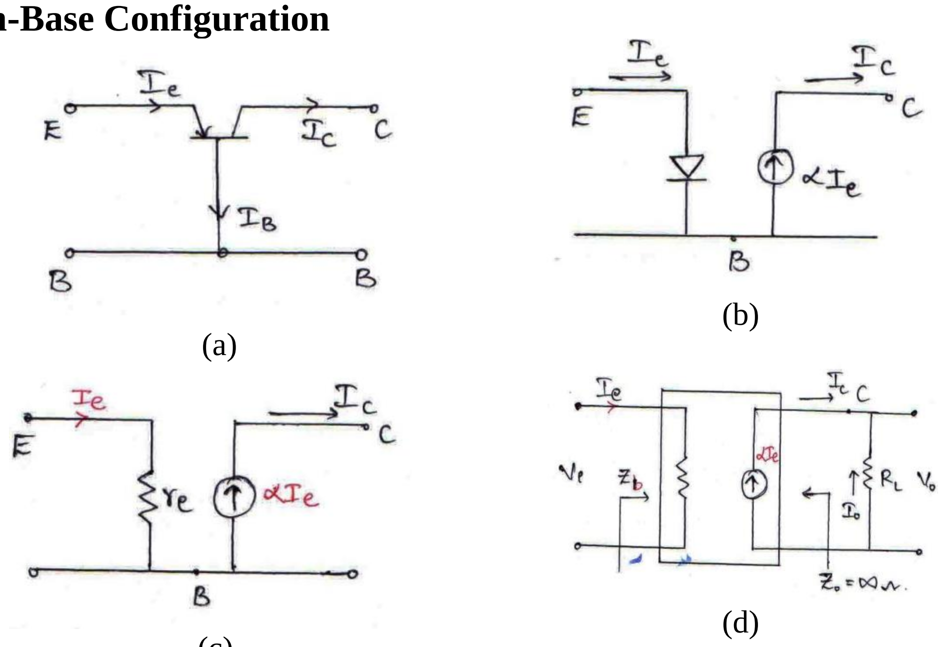


Figure 1 Common Base transistor re mode

We know that from diode equation r_e is defined as follows

$$I_c = \alpha I_e$$
$$r_e = \frac{26\,\mathrm{mV}}{I_e}$$

Applying KVL to input and out circuit of figure 1(d), we will get

input impedance: $Z_i = r_e$

Output impedance: $Z_o = \infty$

Voltage gain: $A_v = \frac{\alpha R_L}{r_e} = \frac{R_L}{r_e}$

Current gain: $A_i = -\alpha = -1$

Common-Emitter Configuration

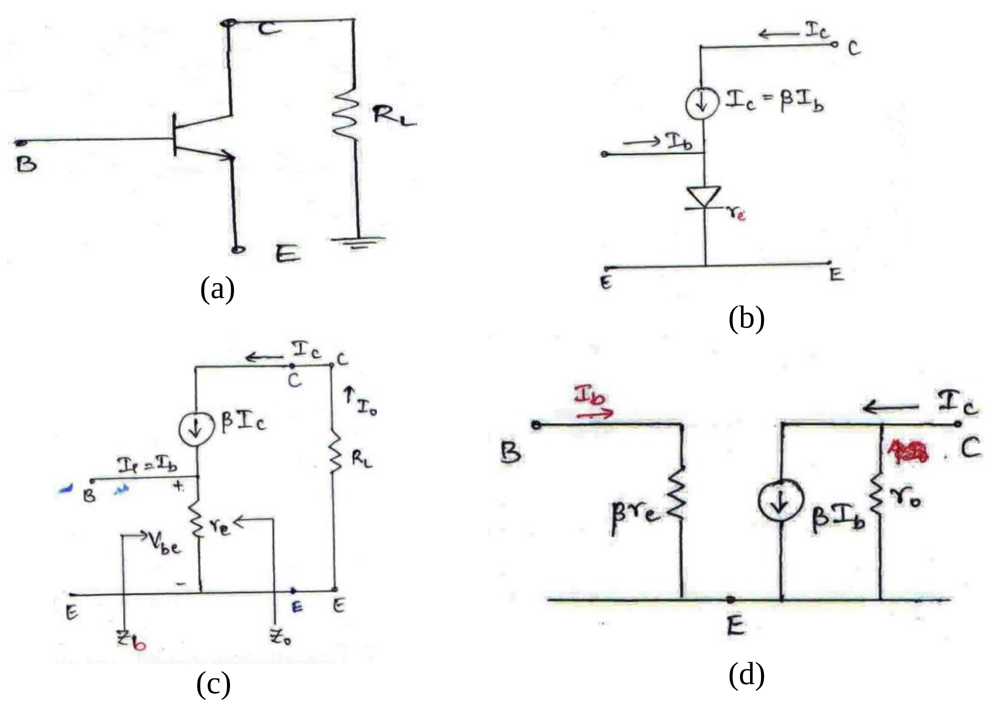


Figure 2 Common Emitter re model of npn transistor

Figure 1 (a) shows simple transistor circuit. Figure 1(b) and 1(c) shows evaluation transistor re model in CE configuration.

Applying KVL to input and out circuit of figure 2(d), we will get

input impedance: $\boldsymbol{z_i = \frac{V_i}{I_i}}$

$V_i = V_{be} = I_e r_e = \beta I_i r_e$

$Z_i = r_e$

Output impedance: $Z_o = \infty$

Voltage gain:

$Vo = -IoRL = -(Ic)RL = -\beta IbRL$

$V_i = I_i Z_i = I_b \beta r_e$

$$A_v = \frac{V_o}{V_i} = -\frac{\beta I_b R_L}{I_b \beta r_e}$$

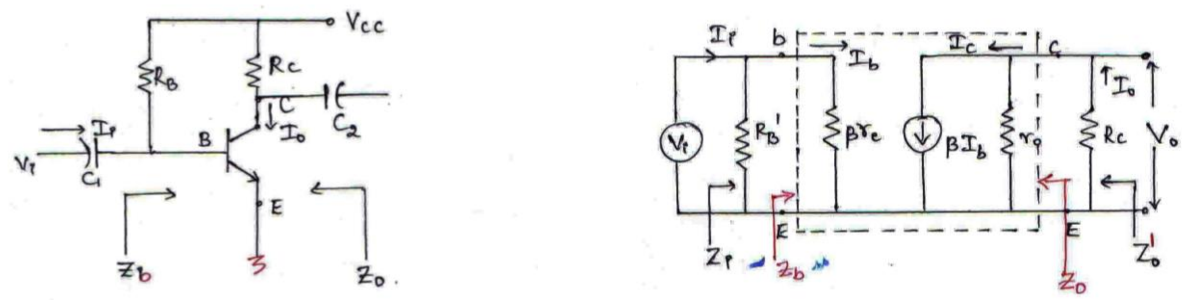
$$A_v = -\frac{R_L}{r_e}$$

Current gain,

$$A_i = \frac{Io}{Ii} = \frac{Ic}{Ib} = \frac{\beta Ib}{Ib}$$

$$A_i = \beta$$

Fixed bias Common-Emitter Configuration



(a) (b)

Figure 3 Fixed bias Common-Emitter Configuration

Note in Fig. 3 (a) that the common ground of the dc supply and the transistor emitter terminal permits the relocation of R_B and R_C in parallel with the input and output sections of the transistor, respectively. In addition, note the placement of the important network parameters Z_i, Z_o, I_i, and I_o on the redrawn network. Substituting the r_e model for the common-emitter configuration of Fig. 3(a) will result in the network of Fig. 3(b).

• From the above r_e model,

Input impedance

$Z_i = [R_B \,||\, \beta r_e]$ ohms

If $R_B > 10\ \beta r_e$, then,

$[R_B \,||\, \beta r_e] \cong \beta r_e$

Then, $Z_i \cong \beta r_e$

Output impedance

Z_o is the output impedance when $V_i = 0$. When $V_i = 0$, $i_b = 0$, resulting in open circuit equivalence for the current source.

$Z_o = [R_C \,||\, r_o]$ ohms

Voltage gain

$V_o = -\beta I_b(R_C \,||\, r_o)$

• From the r_e model, $I_b = V_i / \beta r_e$

• thus,

– $V_o = -\beta (V_i / \beta r_e)(R_C \,||\, r_o)$

– $A_V = V_o / V_i = -(R_C \,||\, r_o) / r_e$

10

• If $r_o > 10R_C$,

– $A_V = -(R_C / r_e)$

• The negative sign in the gain expression indicates that there exists 180o phase shift between the input and output.

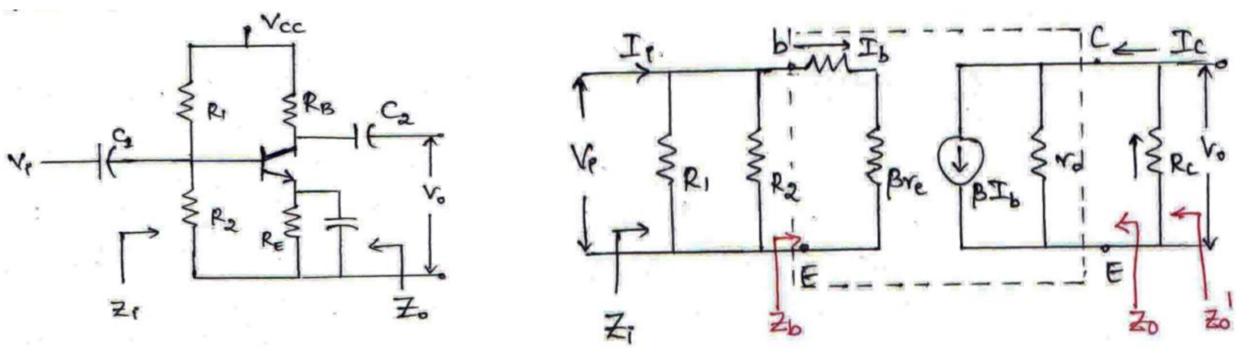
Current gain:

$$A_i = \frac{I_o}{I_i} = \frac{\beta R_B r_o}{(r_o + R_C)(R_B + \beta r_e)}$$

$$A_i \cong \beta \Big|_{r_o \geq 10R_C,\ R_B \geq 10\beta r_e}$$

$$A_i = -A_v \frac{Z_i}{R_C}$$

Common-Emitter Voltage-Divider Bias



(a) (b)

Figure 4 Voltage Divider bias Common-Emitter Configuration

The r_e model is very similar to the fixed bias circuit except for R_B is $R_1 || R_2$ in the case of voltage divider bias.

Input impedance:

$$Z_b = \beta r_e$$
$$R_B = R_1 || R_2$$
$$Z_i = R_B || \beta r_e$$

Output impedance:

$$Z_o = r_o$$
$$Z_o' = R_C || r_o$$
$$Z_o{}' = R_C | r_{o \gg 10 R_C}$$

Voltage gain: From the r_e model, $I_b = V_i / \beta r_e$ thus,

$$V_o = -\beta (V_i / \beta r_e) (R_C || r_o)$$

$$A_v = \frac{V_o}{V_i} = \frac{-R_C || r_o}{r_e}$$

$$A_v = \frac{V_o}{V_i} \cong \frac{-R_C}{r_e} | r_o \geq 10 R_C$$

Current gain:

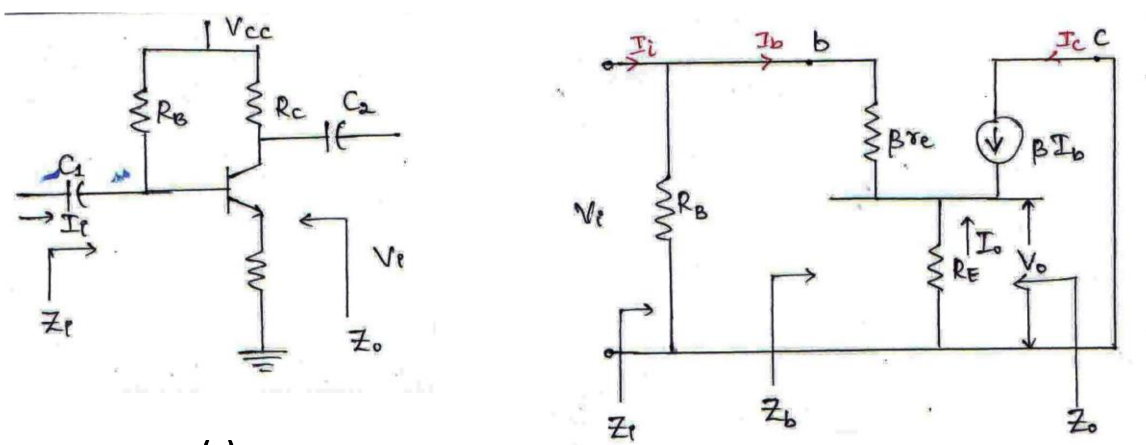
$$A_i = \frac{I_o}{I_i} = \frac{\beta R_B r_o}{(r_o + R_C)(R_B + \beta r_e)}$$

$$A_i = \frac{I_o}{I_i} = \frac{\beta R_B}{(R_B + \beta r_e)} \quad \text{if} \quad r_o \geq 10 R_C$$

$$A_i = \frac{I_o}{I_i} = \beta \quad \text{if} \quad R_B \geq 10 \beta r_e$$

$$A_i = -A_v \frac{Z_i}{R_C}$$

Common-Emitter Emitter-Bias Configuration



(a)

(b)

Figure 4 Fixed bias Common-Emitter Configuration with un bypassed R_E

Input impedance:

Applying KVL to the input side:

$$V_i = I_b \beta r_e + I_e R_E$$

$$V_i = I_b \beta r_e + (\beta + 1) I_b R_E$$

Input impedance looking into the network to the right of RB is

$$Z_b = V_i / I_b = \beta r_e + (\beta + 1) R_E$$

Since β>>1, (β +1) = β

$$Z_b = V_i / I_b = \beta (r_e + R_E)$$

Since R_E is often much greater than r_e,

$$Z_b = \beta R_E,$$

$$Z_i = R_B || Z_b$$

Output impedance: Z_o is determined by setting V_i to zero, $I_b = 0$ and β I_b can be replaced by open circuit equivalent. The result is,

$$Z_o = R_C$$

Voltage gain:

We know that, $V_o = - I_o R_C$

$= - \beta I_b R_C$

$= - \beta(V_i/Z_b)R_C$

$A_V = V_o / V_i = - \beta[R_C /(r_e + R_E)]$

$R_E >> r_e$, $A_V = V_o / V_i = - \beta[R_C /R_E]$

$$A_v = \frac{V_o}{V_i} = \frac{-R_C||r_o}{Z_b}$$

Substituting, $Z_b = \beta(r_e + R_E)$

$$A_v = \frac{V_o}{V_i} \cong \frac{-R_C}{r_e + R_E}|Z_b = \beta(r_e + R_E)$$
$$A_v = \frac{V_o}{V_i} \cong \frac{-R_C}{R_E}|(r_e \ll R_E)$$

Phase relation: The negative sign in the gain equation reveals a 180^o phase shift between input and output.

Current gain:

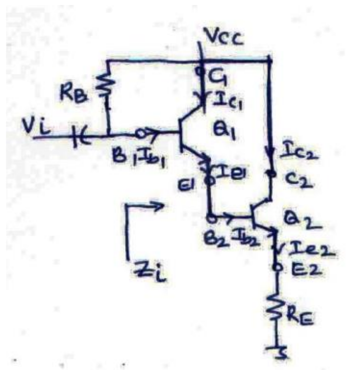
$$A_i = \frac{I_o}{I_i} = \frac{\beta R_B}{(R_B + Z_b)}$$

$$A_i = -A_v \frac{Z_i}{R_C}$$

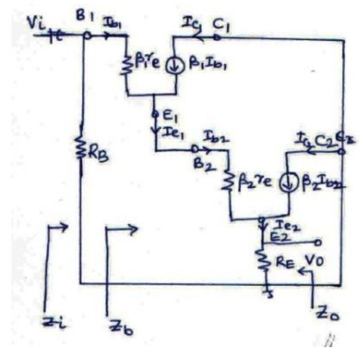
Darlington Emitter Follower

This is also known as the common-collector configuration.

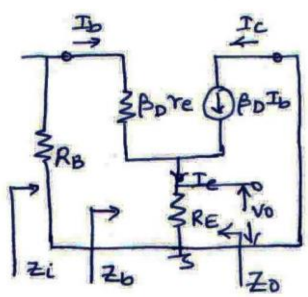
• The input is applied to the base and the output is taken from the emitter. There is no phase shift between input and output.



(a)



(b)



(c)

Figure 5 Darlington Emitter Follower

Input impedance:

$$Z_i = R_B \,||\, Z_b$$
$$Z_b = \beta_D r_e + (\beta_D + 1)R_E$$
$$Z_b = \beta_D \,(r_e + R_E)$$

Since R_E is often much greater than r_e,

$$Z_i = R_B || \beta r_e$$
$$Z_b = \beta_D(r_e + R_E)$$

Output impedance:

To find Zo, it is required to find output equivalent circuit of the emitter follower at its input terminal.

This can be done by writing the equation for the current Ib.

$$I_b = V_i / Z_b$$
$$I_e = (\beta_D + 1)I_b$$
$$= (\beta_D + 1)\,(V_i / Z_b)$$

We know that, $Z_b = \beta_D r_e + (\beta_D + 1)R_E$ substituting this in the equation for Ie we get,

$$I_e = (\beta_D + 1)\,(V_i / Z_b) = (\beta_D + 1)\,(V_i / \beta_D r_e + (\beta_D + 1)R_E\,)$$

$$I_e = V_i / [\beta_D r_e / (\beta_D + 1)] + R_E$$

Since $(\beta_D + 1) = \beta_D$,

$$I_e = V_i / [r_e + R_E]$$

Using the equation $I_e = V_i / [r_e + R_E]$, we can write the output equivalent circuit as,

$$Z_o = R_E || \frac{\beta_D r_e}{\beta_D + 1}$$

$$Z_o \approx R_E || r_e \qquad \text{if } \beta_D \gg 1$$

Since R_E is typically much greater than r_e, $Z_o \approx r_e$

Voltage gain:

Using voltage divider rule for the equivalent circuit,

$$V_o = V_i R_E / (R_E + r_e)$$

$$A_V = V_o / V_i = [R_E / (R_E + r_e)]$$

Since $(R_E + r_e) \cong R_E$,

$A_V \cong [R_E / (R_E] \cong 1$

Phase relationship As seen in the gain equation, output and input are in phase

Current gain:

$$A_i = \frac{I_o}{I_i} = \frac{\beta R_B}{(R_B + Z_b)}$$

H – Parameter model :-

→ The equivalent circuit of a transistor can be dram using simple approximation by retaining its essential features.

→ These equivalent circuits will aid in analyzing transistor circuits easily and rapidly.

Two port devices & Network Parameters:-

→ A transistor can be treated as a two part network. The terminal behaviour of any two part network can be specified by the terminal voltages V_1 & V_2 at parts 1 & 2 respectively and current i_1 and i_2, entering parts 1 & 2, respectively, as shown in figure.

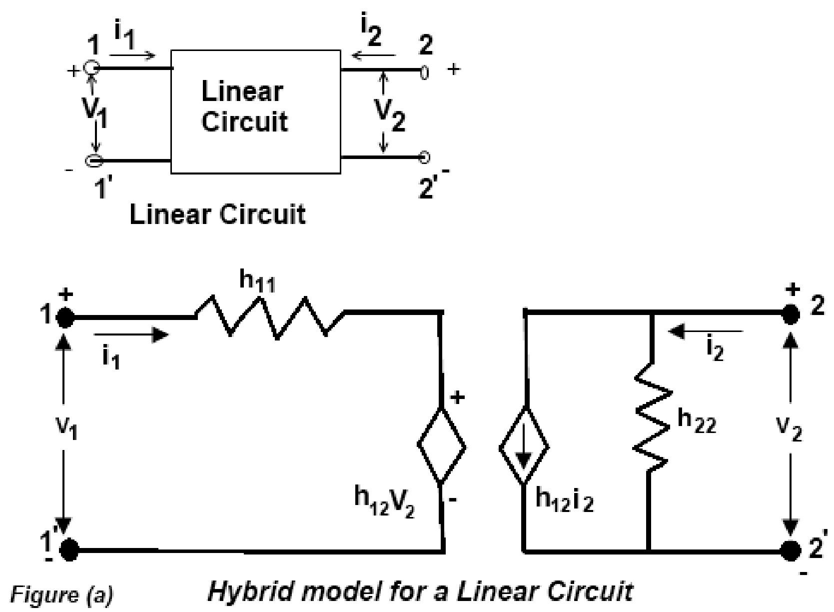


Figure 6 Two port Network

Hybrid parameters (or) h – parameters:-

If the input current i_1 and output Voltage V_2 are takes as independent variables, the input voltage V_1 and output current i_2 can be written as

$V_1 = h_{11}\, i_1 + h_{12}\, V_2$

$i_2 = h_{21} i_1 + h_{22} V_2$

The four hybrid parameters h_{11}, h_{12}, h_{21} and h_{22} are defined as follows.

$h_{11} = [V_1 / i_1]$ with $V_2 = 0$	Input Impedance with output part short circuited.
$h_{22} = [i_2 / V_2]$ with $i_1 = 0$	Output admittance with input part open circuited.
$h_{12} = [V_1 / V_2]$ with $i_1 = 0$	reverse voltage transfer ratio with input part open circuited.
$h_{21} = [i_2 / i_1]$ with $V_2 = 0$	Forward current gain with output part short circuited.

The dimensions of h – parameters are as follows:

h_{11} - Ω

h_{22} – mhos

h_{12}, h_{21} – dimension less.

as the dimensions are not alike, (ie) they are hybrid in nature, and these parameters are called as hybrid parameters.

i= 11 = input ; o = 22 = output ;

f = 21 = forward transfer ; r = 12 = Reverse transfer.

Notations used in transistor circuits:-

$h_i = h_{11}$ = Short circuit input impedance

$h_0 = h_{22}$ = Open circuit output admittance

$h_r = h_{12}$ = Open circuit reverse voltage transfer ratio

$h_f = h_{21}$= Short circuit forward current Gain.

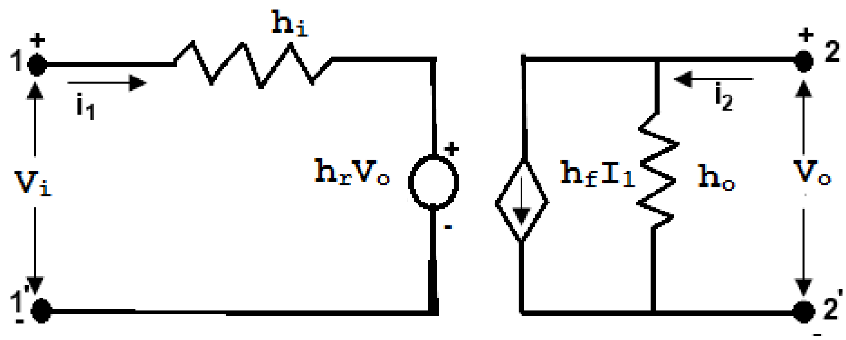
The Hybrid Model for Two-port Network:-

$V_1 = h_{11} i_1 + h_{12} V_2$

$I_2 = h_1 i_1 + h_{22} V_2$

$V_1 = h_1 i_1 + h_r V_2$

$I_2 = h_f\, i_1 + h_0\, V_2$



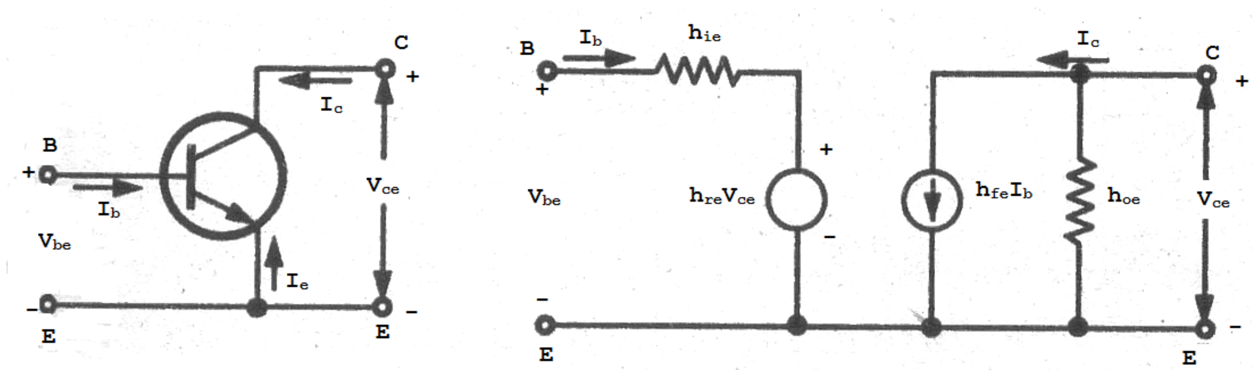
<u>Transistor Hybrid model</u>:-

Essentially, the transistor model is a three terminal two – port system.
The h – parameters, however, will change with each configuration.
To distinguish which parameter has been used or which is available, a second subscript has been added to the h – parameter notation.
For the common – base configuration, the lowercase letter b is added, and for common emitter and common collector configurations, the letters e and c are used respectively.

Normally $h_r$is a relatively small quantity, its removal is approximated by h_r and $h_r V_o = 0$, resulting in a short – circuit equivalent.

The resistance determined by $1/h_o$ is often large enough to be ignored in comparison to a parallel load, permitting its replacement by an open – circuit quivalent.

<u>CE Transistor Circuit</u>



To Derive the Hybrid model for transistor consider the CE circuit shown in figure.The variables are i_B, i_c, $v_{B(=V_{BE})}$ and $v_{c(=V_{CE})}$. i_B and v_c are considered as independent variables.

Then , $v_B = f_1(i_B, v_c)$ ----------------------(1)

$i_C = f_2(i_B, v_c)$ ----------------------(2)

Making a Taylor's series expansion around the quiescent point I_B, V_C and neglecting higher order terms, the following two equations are obtained.

$\Delta v_B = (\partial f_1/\partial i_B)V_c \,.\, \Delta i_B + (\partial f_1/\partial v_c)I_B \,.\, \Delta v_C$ ---------------(3)

$\Delta i_C = (\partial f_2/\partial i_B)V_c \,.\, \Delta i_B + (\partial f_2/\partial v_c)I_B \,.\, \Delta v_C$ ----------------(4)

The partial derivatives are taken keeping the collector voltage or base current constant as indicated by the subscript attached to the derivative.

Δv_B , Δv_C , Δi_C , Δi_B represent the small signal(increment) base and collector voltages and currents,they are represented by symbols v_b , v_c , i_b and i_c respectively.

Eqs (3) and (4) may be written as

$V_b = h_{ie}\, i_b + h_{re}\, V_c$

$i_c = h_{fe}\, i_b + h_{oe}\, V_c$

Where $h_{ie} = (\partial f_1/\partial i_B)V_c = (\partial v_B/\partial i_B)V_c = (\Delta v_B / \Delta i_B)V_c = (v_b / i_b)V_c$

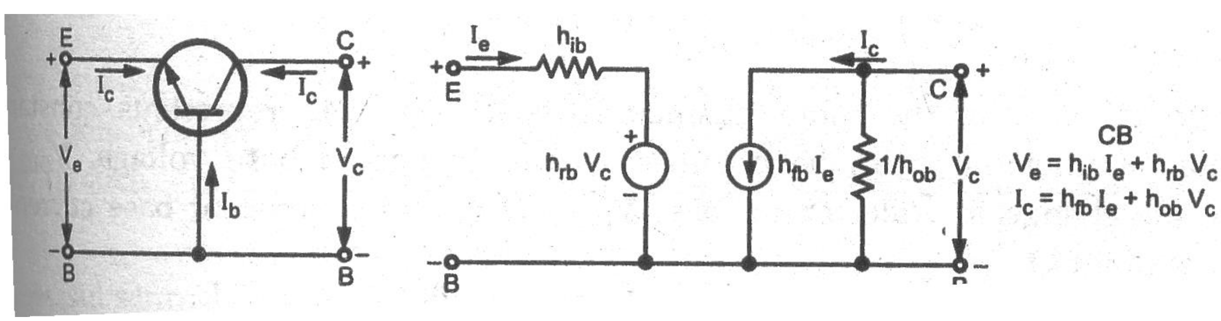
$h_{re} = (\partial f_1/\partial v_c)I_B = (\partial v_B/\partial v_c)\, I_B = (\Delta v_B / \Delta v_c)\, I_B = (v_b / v_c)\, I_B$

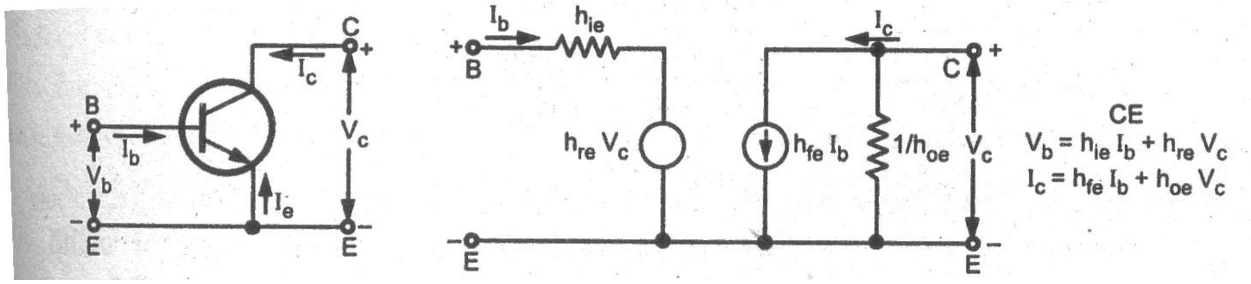
$h_{fe} = (\partial f_2/\partial i_B)V_c = (\partial i_c / \partial i_B)V_c = (\Delta i_c / \Delta i_B)V_c = (i_c / i_b)V_c$

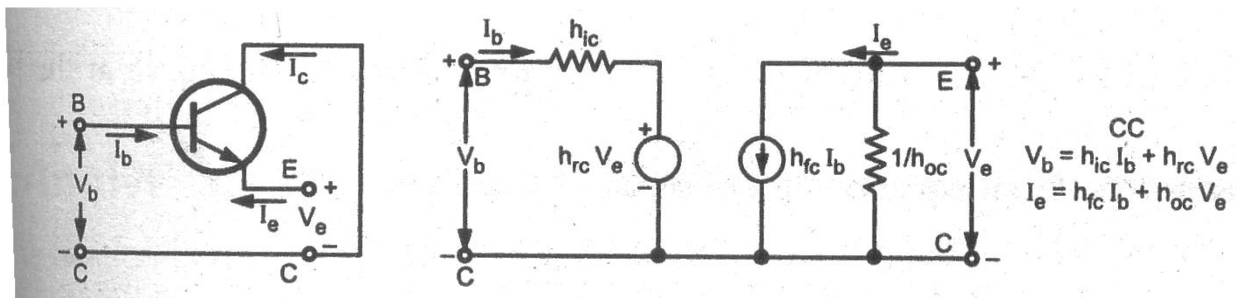
$h_{oe} = (\partial f_2/\partial v_c)I_B = (\partial i_c / \partial v_c)\, I_B = (\Delta i_c / \Delta v_c)\, I_B = (i_c / v_c)\, I_B$

The above equations define the h-parameters of the transistor in CE configuration.The same theory can be extended to transistors in other configurations.

Hybrid Model and Equations for the transistor in three different configurations are are given below.



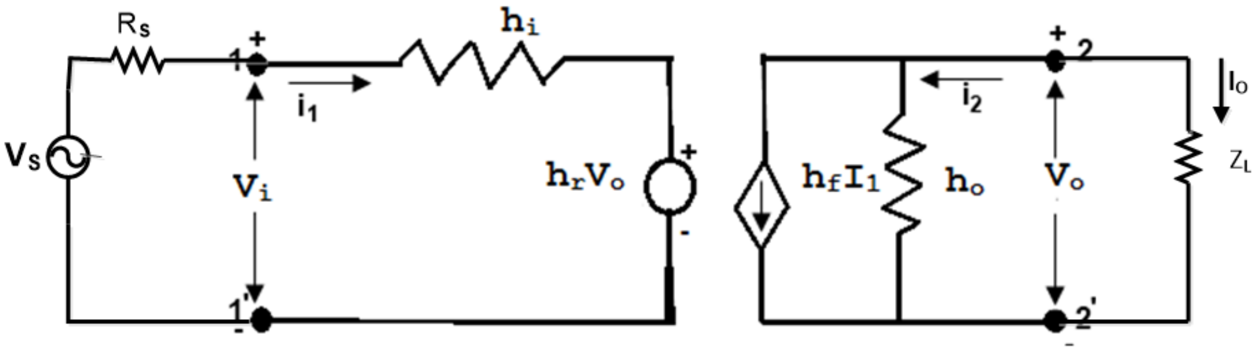




Comparision of H parameters

CB	CE	CC
$h_{ib} = \frac{v_{eb}}{i_e}$	$h_{ie} = \frac{v_{be}}{i_b}$	$h_{ic} = \frac{v_{bc}}{i_b}$
$h_{rb} = \frac{v_{eb}}{v_{cb}}$	$h_{re} = \frac{v_{be}}{v_{ce}}$	$h_{rc} = \frac{v_{bc}}{v_{ec}}$
$h_{fb} = \frac{i_c}{i_e}$	$h_{fe} = \frac{i_c}{i_b}$	$h_{fc} = \frac{i_e}{i_b}$
$h_{ob} = \frac{i_c}{v_{cb}}$	$h_{oe} = \frac{i_c}{v_{ce}}$	$h_{oc} = \frac{i_e}{v_{ec}}$

Analysis of transistor amplifier using h parameters.



For analysis of transistor amplifier we have to determine the following terms:

- Current Gain
- Voltage gain
- Input impedance
- Output impedance

Current gain:

For the transistor amplifier stage, A_i is defined as the ratio of output to input currents.

$$A_i = \frac{I_L}{I_b} = \frac{-I_C}{I_b} \qquad (I_L + I_c = 0. \quad \therefore I_L = -I_c)$$

$$I_C = h_{fe} I_b + h_{oe} V_c$$

$$V_c = I_L Z_L = -I_c Z_L$$

$$\therefore I_c = h_{fe} I_b + h_{oe} (-I_c Z_L)$$

$$\text{or} \quad \frac{I_c}{I_b} = \frac{h_{fe}}{1 + h_{oe} Z_L}$$

$$\therefore A_i = -\frac{h_{fe}}{1 + h_{oe} Z_L}$$

Input Impedence:

The impedence looking into the amplifier input terminals (1,1') is the input impedence Z_i

$$Z_i = \frac{V_b}{I_b}$$

$$V_b = h_{ie} I_b + h_{re} V_c$$

$$\frac{V_b}{I_b} = h_{ie} + h_{re} \frac{V_c}{I_b}$$

$$= h_{ie} - \frac{h_{re} I_c Z_L}{I_b}$$

$$\therefore Z_i = h_{ie} + h_{re} A_I Z_L$$

$$= h_{ie} - \frac{h_{re} h_{fe} Z_L}{1 + h_{oe} Z_L}$$

$$\therefore Z_i = h_{ie} - \frac{h_{re} h_{fe}}{Y_L + h_{oe}} \qquad (\text{since } Y_L = \frac{1}{Z_L})$$

Voltage gain:

The ratio of output voltage to input voltage gives the gain of the transistors.

$$A_v = \frac{V_C}{V_b} = -\frac{I_C Z_L}{V_b}$$

$$\therefore A_v = \frac{I_B A_i Z_L}{V_b} = \frac{A_i Z_L}{Z_i}$$

Output Admittance: It is defined

$$Y_0 = \left.\frac{I_c}{V_c}\right|_{V_s = 0}$$

$$I_c = h_{fe} I_b + h_{oe} V_c$$

$$\frac{I_c}{V_c} = h_{fe}\frac{I_b}{V_c} + h_{oe}$$

$$\text{when } V_s = 0, \quad R_s . I_b + h_{ie} . I_b + h_{re} V_c = 0.$$

$$\frac{I_b}{V_c} = -\frac{h_{re}}{R_s + h_{ie}}$$

$$\therefore Y_0 = h_{oe} - \frac{h_{re}\, h_{fe}}{R_s + h_{ie}}$$

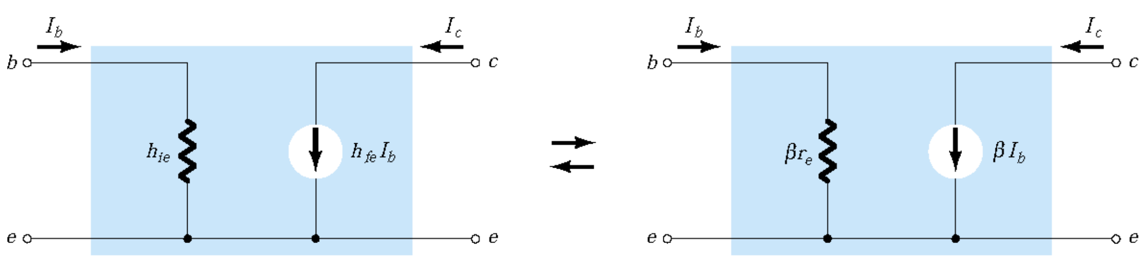
Voltage amplification taking into account source impedance (R_S) is given by

$$A_{VS} = \frac{V_c}{V_s} = \frac{V_c}{V_b} * \frac{V_b}{V_s} \qquad \left(V_b = \frac{V_s}{R_s + Z_i} * Z_i\right)$$

$$= A_V . \frac{Z_i}{Z_i + R_s}$$

$$= \frac{A_i\, Z_L}{Z_i + R_s}$$

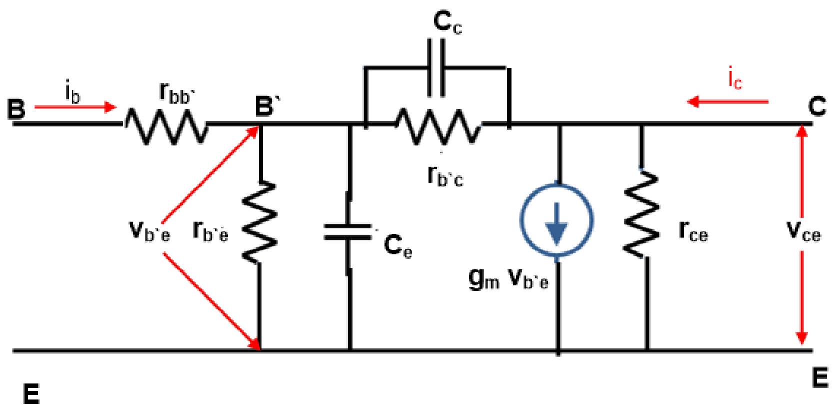
Simplified Hybrid model is identical to the re model is as shown in fig. refer re model analysis



Hybrid versus *re* model: (a) common-emitter configuration

Hybrid ⋅ *π* model

- The hybrid-pi or Giacoletto model of common emitter transistor model is given below. The resistance components in this circuit can be obtained from the low frequency hparameters.
- For high frequency analysis transistor is replaced by high frequency hybrid-pi model and voltage gain, current gain and input impedance are determined.



This is more accurate model for high frequency effects. The capacitors that appear are stray parasitic capacitors between the various junctions of the device. These capacitances come into picture only at high frequencies.

- C_{bc} or C_u is usually few pico farads to few tens of pico farads.
- r_{bb} includes the base contact, base bulk and base spreading resistances.
- r_{be} (r_π), r_{bc}, r_{ce} are the resistances between the indicated terminals.
- r_{be} (r_π) is simply βr_e introduced for the CE r_e model.
- r_{bc} is a large resistance that provides feedback between the output and the input.
- $r_\pi = \beta r_e$
- $g_m = 1/r_e$
- $r_o = 1/h_{oe}$
- $h_{re} = r_\pi / (r_\pi + r_{bc})$

The **transconductance, gm**, is related to the dynamic (differential) resistance, re, of the forward-biased emitter-base junction:

$$
\begin{aligned}
g_m &= \partial Ic/\partial Vb'e \\
&= \alpha \partial Ie/\partial Vb'e \\
&\approx \alpha/re \\
&\approx Ic/Vth
\end{aligned}
$$

Vth = kBT/q

The resistance *rbb'* is the base spreading resistance.

The resistance *rb'c* and the capacitance *Cb'c (Cc*) represent the dynamic (differential) resistance and the capacitance of the reverse-biased collector-base junction.

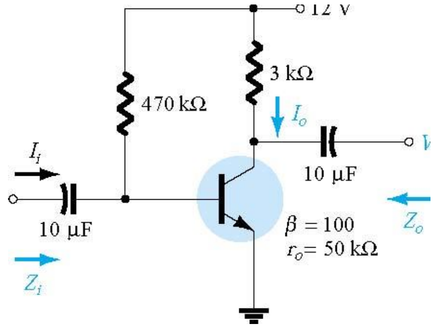
Using transconductance:

$$i_c \approx g_m v_{b'e}$$

(ignoring the current through r_{ce})

Example 1

(a) Determine r_e. (b) Find Z_i (c) Calculate Z_o (d) Determine A_v (e) Find A_i (f) Repeat parts (c) through (e) including $r_o = 50\text{ k}\Omega$ in all calculations and compare results. (From Text Book - Boylestad)



Solution

(a) DC analysis:

$$I_B = \frac{V_{CC} - V_{BE}}{R_B} = \frac{12\text{ V} - 0.7\text{ V}}{470\text{ k}\Omega} = 24.04\ \mu\text{A}$$

$$I_E = (\beta + 1)I_B = (101)(24.04\ \mu\text{A}) = 2.428\text{ mA}$$

$$r_e = \frac{26\text{ mV}}{I_E} = \frac{26\text{ mV}}{2.428\text{ mA}} = \mathbf{10.71\ \Omega}$$

(b) $\beta r_e = (100)(10.71\ \Omega) = 1.071\text{ k}\Omega$

$Z_i = R_B \| \beta r_e = 470\text{ k}\Omega \| 1.071\text{ k}\Omega = \mathbf{1.069\text{ k}\Omega}$

(c) $Z_o = R_C = \mathbf{3\text{ k}\Omega}$

(d) $A_v = -\dfrac{R_C}{r_e} = -\dfrac{3\text{ k}\Omega}{10.71\ \Omega} = \mathbf{-280.11}$

(e) Since $R_B \geq 10\beta r_e (470\text{ k}\Omega > 10.71\text{ k}\Omega)$

$A_i \cong \beta = \mathbf{100}$

(f) $Z_o = r_o \| R_C = 50\text{ k}\Omega \| 3\text{ k}\Omega = \mathbf{2.83\text{ k}\Omega}$ vs. $3\text{ k}\Omega$

$$A_v = -\frac{r_o \| R_C}{r_e} = \frac{2.83\text{ k}\Omega}{10.71\ \Omega} = \mathbf{-264.24} \text{ vs. } -280.11$$

$$A_i = \frac{\beta R_B r_o}{(r_o + R_C)(R_B + \beta r_e)} = \frac{(100)(470\text{ k}\Omega)(50\text{ k}\Omega)}{(50\text{ k}\Omega + 3\text{ k}\Omega)(470\text{ k}\Omega + 1.071\text{ k}\Omega)}$$

$$= \mathbf{94.13} \text{ vs. } 100$$

As a check:

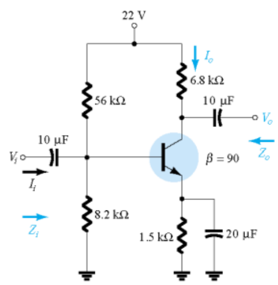
$$A_i = -A_v \frac{Z_i}{R_C} = \frac{-(-264.24)(1.069\text{ k}\Omega)}{3\text{ k}\Omega} = \mathbf{94.16}$$

which differs slightly only due to the accuracy carried through the calculations.

Example 2

For the network of Figure, determine:

(a) r_e.
(b) Z_i.
(c) Z_o ($r_o = \infty\ \Omega$).
(d) A_v ($r_o = \infty\ \Omega$).
(e) A_i ($r_o = \infty\ \Omega$).
(f) The parameters of parts (b) through (e) if $r_o = 1/h_{oe} = 50\text{ k}\Omega$ and compare results.



Solution

(a) DC: Testing $\beta R_E > 10R_2$

$$(90)(1.5\text{ k}\Omega) > 10(8.2\text{ k}\Omega)$$
$$135\text{ k}\Omega > 82\text{ k}\Omega \ (\textit{satisfied})$$

Using the approximate approach,

$$V_B = \frac{R_2}{R_1 + R_2}V_{CC} = \frac{(8.2\text{ k}\Omega)(22\text{ V})}{56\text{ k}\Omega + 8.2\text{ k}\Omega} = 2.81\text{ V}$$
$$V_E = V_B - V_{BE} = 2.81\text{ V} - 0.7\text{ V} = 2.11\text{ V}$$

$$I_E = \frac{V_E}{R_E} = \frac{2.11\text{ V}}{1.5\text{ k}\Omega} = 1.41\text{ mA}$$
$$r_e = \frac{26\text{ mV}}{I_E} = \frac{26\text{ mV}}{1.41\text{ mA}} = \mathbf{18.44\ \Omega}$$

(b) $R' = R_1 \| R_2 = (56\text{ k}\Omega)\|(8.2\text{ k}\Omega) = 7.15\text{ k}\Omega$

$Z_i = R' \| \beta r_e = 7.15\text{ k}\Omega \| (90)(18.44\ \Omega) = 7.15\text{ k}\Omega \| 1.66\text{ k}\Omega$
$= \mathbf{1.35\text{ k}\Omega}$

(c) $Z_o = R_C = \mathbf{6.8\text{ k}\Omega}$

(d) $A_v = -\dfrac{R_C}{r_e} = -\dfrac{6.8\text{ k}\Omega}{18.44\ \Omega} = \mathbf{-368.76}$

(e) The condition $R' \geq 10\beta r_e$ ($7.15\text{ k}\Omega \geq 10(1.66\text{ k}\Omega) = 16.6\text{ k}\Omega$ is *not* satisfied. Therefore,

$$A_i \cong \frac{\beta R'}{R' + \beta r_e} = \frac{(90)(7.15\text{ k}\Omega)}{7.15\text{ k}\Omega + 1.66\text{ k}\Omega} = \mathbf{73.04}$$

(f) $Z_i = \mathbf{1.35\text{ k}\Omega}$

$Z_o = R_C \| r_o = 6.8\text{ k}\Omega \| 50\text{ k}\Omega = \mathbf{5.98\text{ k}\Omega}$ vs. $6.8\text{ k}\Omega$

$A_v = -\dfrac{R_C \| r_o}{r_e} = -\dfrac{5.98\text{ k}\Omega}{18.44\ \Omega} = \mathbf{-324.3}$ vs. -368.76

The condition

$$r_o \geq 10R_C \ (50\text{ k}\Omega \geq 10(6.8\text{ k}\Omega) = 68\text{ k}\Omega)$$

is *not* satisfied. Therefore,

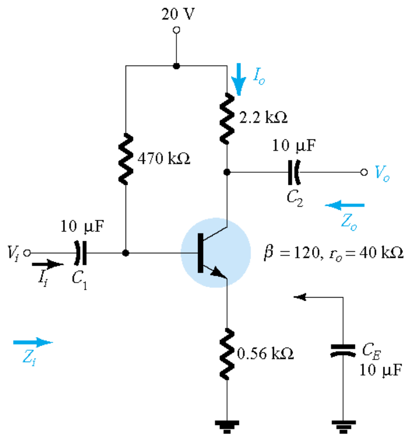
$$A_i = \frac{\beta R' r_o}{(r_o + R_C)(R' + \beta r_e)} = \frac{(90)(7.15\text{ k}\Omega)(50\text{ k}\Omega)}{(50\text{ k}\Omega + 6.8\text{ k}\Omega)(7.15\text{ k}\Omega + 1.66\text{ k}\Omega)}$$
$$= \mathbf{64.3} \text{ vs. } 73.04$$

There was a measurable difference in the results for Z_o, A_v, and A_i because condition $r_o \geq 10R_C$ was *not* satisfied.

Example 3

For the network of Fig. , without C_E (unbypassed), determine:

(a) r_e.
(b) Z_i.
(c) Z_o.
(d) A_v.
(e) A_i.



Solution

(a) DC: $I_B = \dfrac{V_{CC} - V_{BE}}{R_B + (\beta + 1)R_E} = \dfrac{20\text{ V} - 0.7\text{ V}}{470\text{ k}\Omega + (121)0.56\text{ k}\Omega} = 35.89\ \mu\text{A}$

$I_E = (\beta + 1)I_B = (121)(46.5\ \mu\text{A}) = 4.34\text{ mA}$

and $\quad r_e = \dfrac{26\text{ mV}}{I_E} = \dfrac{26\text{ mV}}{4.34\text{ mA}} = \mathbf{5.99\ \Omega}$

(b) Testing the condition $r_o \geq 10(R_C + R_E)$,

$$40\text{ k}\Omega \geq 10(2.2\text{ k}\Omega + 0.56\text{ k}\Omega)$$

$$40\text{ k}\Omega \geq 10(2.76\text{ k}\Omega) = 27.6\text{ k}\Omega\ (\textit{satisfied})$$

Therefore,

$$Z_b \cong \beta(r_e + R_E) = 120(5.99\ \Omega + 560\ \Omega) = 67.92\text{ k}\Omega$$

and

$$Z_i = R_B \| Z_b = 470\text{ k}\Omega \| 67.92\text{ k}\Omega = \mathbf{59.34\ k\Omega}$$

(c) $Z_o = R_C = \mathbf{2.2\ k\Omega}$

(d) $r_o \geq 10R_C$ is satisfied. Therefore,

$$A_v = \frac{V_o}{V_i} \cong -\frac{\beta R_C}{Z_b} = -\frac{(120)(2.2\text{ k}\Omega)}{67.92\text{ k}\Omega} = \mathbf{-3.89}$$

compared to -3.93 using Eq. (8.27): $A_v \cong -R_C/R_E$.

(e) $A_i = -A_v \dfrac{Z_i}{R_C} = -(-3.89)\left(\dfrac{59.34\text{ k}\Omega}{2.2\text{ k}\Omega}\right) = \mathbf{104.92}$

compared to 104.85 using Eq. (8.28): $A_i \cong \beta R_B/(R_B + Z_b)$.

Example 4

Repeat the analysis of Example 3 with C_E in place.

Solution

(a) The dc analysis is the same, and $r_e = 5.99\ \Omega$.
(b) R_E is "shorted out" by C_E for the ac analysis. Therefore,

$$Z_i = R_B \| Z_b = R_B \| \beta r_e = 470\ \text{k}\Omega \| (120)(5.99\ \Omega)$$
$$= 470\ \text{k}\Omega \| 718.8\ \Omega \cong \mathbf{717.70\ \Omega}$$

(c) $Z_o = R_C = \mathbf{2.2\ k\Omega}$

(d) $A_v = -\dfrac{R_C}{r_e}$

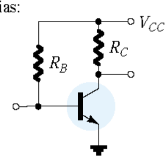
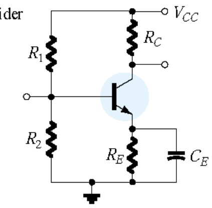
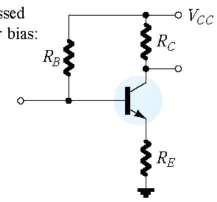
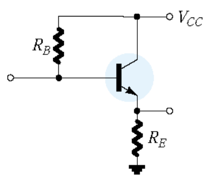
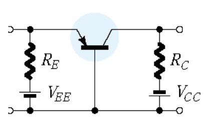
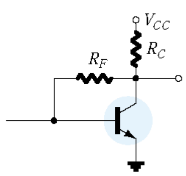
$$= -\frac{2.2\ \text{k}\Omega}{5.99\ \Omega} = \mathbf{-367.28} \quad \text{(a significant increase)}$$

(e) $A_i = \dfrac{\beta R_B}{R_B + Z_b} = \dfrac{(120)(470\ \text{k}\Omega)}{470\ \text{k}\Omega + 718.8\ \Omega}$

$$= \mathbf{119.82}$$

Summary of Transistor small signal analysis

TABLE 8.1 Relative Levels for the Important Parameters of the CE, CB, and CC Transistor Amplifiers

Configuration	Z_i	Z_o	A_v	A_i
Fixed-bias: 	Medium (1 kΩ) $= \boxed{R_B \| \beta r_e}$ $\cong \boxed{\beta r_e}$ $(R_B \geq 10\beta r_e)$	Medium (2 kΩ) $= \boxed{R_C \| r_o}$ $\cong \boxed{R_C}$ $(r_o \geq 10R_C)$	High (−200) $= \boxed{-\frac{(R_C \| r_o)}{r_e}}$ $\cong \boxed{-\frac{R_C}{r_e}}$ $(r_o \geq 10R_C)$	High (100) $= \boxed{\frac{\beta R_B r_o}{(r_o + R_C)(R_B + \beta r_e)}}$ $\cong \boxed{\beta}$ $(r_o \geq 10R_C,$ $R_B \geq 10\beta r_e)$
Voltage-divider bias: 	Medium (1 kΩ) $= \boxed{R_1 \| R_2 \| \beta r_e}$	Medium (2 kΩ) $= \boxed{R_C \| r_o}$ $\cong \boxed{R_C}$ $(r_o \geq 10R_C)$	High (−200) $= \boxed{-\frac{R_C \| r_o}{r_e}}$ $\cong \boxed{-\frac{R_C}{r_e}}$ $(r_o \geq 10R_C)$	High (50) $= \boxed{\frac{\beta(R_1 \| R_2) r_o}{(r_o + R_C)(R_1 \| R_2 + \beta r_e)}}$ $\cong \boxed{\frac{\beta(R_1 \| R_2)}{R_1 \| R_2 + \beta r_e}}$ $(r_o \geq 10R_C)$
Unbypassed emitter bias: 	High (100 kΩ) $= \boxed{R_B \| Z_b}$ $Z_b \cong \beta(r_e + R_E)$ $\cong \boxed{R_B \| \beta R_E}$ $(R_E \gg r_e)$	Medium (2 kΩ) $= \boxed{R_C}$ (any level of r_o)	Low (−5) $= \boxed{-\frac{R_C}{r_e + R_E}}$ $\cong \boxed{-\frac{R_C}{R_E}}$ $(R_E \gg r_e)$	High (50) $\cong \boxed{-\frac{\beta R_B}{R_B + Z_b}}$
Emitter-follower: 	High (100 kΩ) $= \boxed{R_B \| Z_b}$ $Z_b \cong \beta(r_e + R_E)$ $\cong \boxed{R_B \| \beta R_E}$ $(R_E \gg r_e)$	Low (20 Ω) $= \boxed{R_E \| r_e}$ $\cong \boxed{r_e}$ $(R_E \gg r_e)$	Low ($\cong 1$) $= \boxed{\frac{R_E}{R_E + r_e}}$ $\cong \boxed{1}$	High (−50) $\cong \boxed{-\frac{\beta R_B}{R_B + Z_b}}$
Common-base: 	Low (20 Ω) $= \boxed{R_E \| r_e}$ $\cong \boxed{r_e}$ $(R_E \gg r_e)$	Medium (2 kΩ) $= \boxed{R_C}$	High (200) $\cong \boxed{\frac{R_C}{r_e}}$	Low (−1) $\cong \boxed{-1}$
Collector feedback: 	Medium (1 kΩ) $= \boxed{\frac{r_e}{\frac{1}{\beta} + \frac{R_C}{R_F}}}$ $(r_o \geq 10R_C)$	Medium (2 kΩ) $\cong \boxed{R_C \| R_F}$ $(r_o \geq 10R_C)$	High (−200) $\cong \boxed{-\frac{R_C}{r_e}}$ $(r_o \geq 10R_C)$ $R_F \gg R_C)$	High (50) $= \boxed{\frac{\beta R_F}{R_F + \beta R_C}}$ $\cong \boxed{\frac{R_F}{R_C}}$