Module-2

Automatic Generation Control

Introduction

The main objective of power system operation and control is to maintain continuous supply of power with an acceptable quality, to all the consumers in the system. The system will be in equilibrium, when there is a balance between the power demand and the power generated. As the power in AC form has real and reactive components: the real power balance; as well as the reactive power balance is to be achieved.

There are two basic control mechanisms used to achieve reactive power balance (acceptable voltage profile) and real power balance (acceptable frequency values). The former is called the automatic voltage regulator (AVR) and the latter is called the automatic load frequency control (ALFC) or automatic generation control (AGC).

Generator Voltage Control System

The voltage of the generator is proportional to the speed and excitation (flux) of the generator. The speed being constant, the excitation is used to control the voltage. Therefore, the voltage control system is also called as excitation control system or automatic voltage regulator (AVR). For the alternators, the excitation is provided by a device (another machine or a static device) called exciter. For a large alternator the exciter may be required to supply a field current of as large as 6500A at 500V and hence the exciter is a fairly large machine. Depending on the way the dc supply is given to the field winding of the alternator (which is on the rotor), the exciters are classified as: i) DC Exciters; ii) AC Exciters; and

iii) Static Exciters. Accordingly, several standard block diagrams are developed by the IEEE working group to represent the excitation system. A schematic of an excitation control system is shown in Fig2.1.

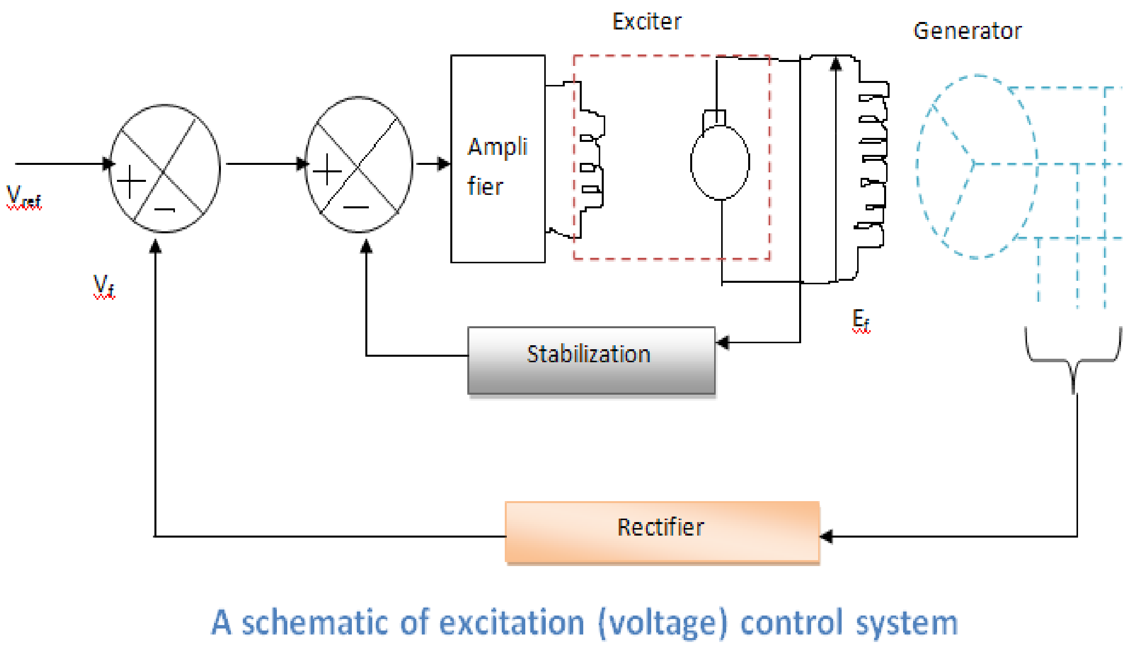
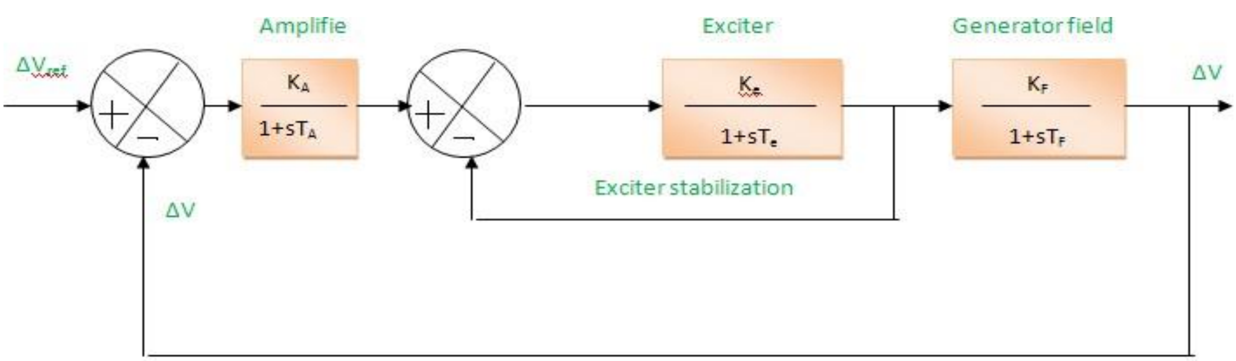


Fig2.1: A schematic of Excitation (Voltage) Control System.

A simplified block diagram of the generator voltage control system is shown in Fig2.2. The generator terminal voltage V_t is compared with a voltage reference V_{ref} to obtain a voltage error signal ΔV. This signal is applied to the voltage regulator shown as a block with transfer function $K_A/(1+T_A s)$. The output of the regulator is then applied to exciter shown with a block of transfer function $K_e/(1+T_e s)$. The output of the exciter E_{fd} is then applied to the field winding which adjusts the generator terminal voltage. The generator field can be represented by a block with a transfer function $K_F/(1+sT_F)$. The total transfer

function is $\frac{\Delta V}{\Delta V_{ref}} = \frac{G(s)}{1+G(s)}$ where, $G(s) = \frac{K_A K_e K_F}{(1+sT_A)(1+sT_e)(1+sT_F)}$

The stabilizing compensator shown in the diagram is used to improve the dynamic response of the exciter. The input to this block is the exciter voltage and the output is a stabilizing feedback signal to reduce the excessive overshoot.



A simplified block diagram of voltage (excitation) control system

Fig2.2: A simplified block diagram of Voltage (Excitation) Control System.

Performance of AVR Loop

The purpose of the AVR loop is to maintain the generator terminal voltage with in acceptable values. A static accuracy limit in percentage is specified for the AVR, so that the terminal voltage is maintained within that value. For example, if the accuracy limit is 4%, then the terminal voltage must be maintained with in 4% of the base voltage.

The performance of the AVR loop is measured by its ability to regulate the terminal voltage of the generator within prescribed static accuracy limit with an acceptable speed of response. Suppose the static accuracy limit is denoted by A_c in percentage with reference to the nominal value. The error voltage is to be less than $(A_c/100)\Delta|V|_{ref}$.

From the block diagram, for a steady state error voltage Δe;

$$\Delta e = \Delta|V|_{ref} - \Delta|V|_t < \frac{Ac}{100}\Delta|V|_{ref}$$

$$\Delta e = \Delta|V|_{ref} - \Delta|V|_t = \Delta|V|_{ref} - \frac{G(s)}{1+G(s)}\Delta|V|_{ref}$$

$$= \left\{1 - \frac{G(s)}{1+G(s)}\right\}\Delta|V|_{ref}$$

For constant input condition, $(s \to 0)$

$$\Delta e = \left\{1 - \frac{G(s)}{1+G(s)}\right\}\Delta|V|_{ref} = \left\{1 - \frac{G(0)}{1+G(0)}\right\}\Delta|V|_{ref}$$

$$= \frac{1}{1+G(0)}\Delta|V|_{ref} = \frac{1}{1+K}\Delta|V|_{ref}$$

where, $K = G(0)$ is the open loop gain of the AVR. Hence,

$$\frac{1}{1+K}\Delta|V|_{ref} < \frac{Ac}{100}\Delta|V|_{ref} \quad \text{or} \quad K > \left\{\frac{100}{Ac} - 1\right\}$$

Example1: Find the open loop gain of an AVR loop if the static accuracy required is 3%.

Solution: Given A_c = 3%. $K > \{ \frac{100}{Ac} - 1\} = K > \{ \frac{100}{3} - 1\} = 32.33$. Thus, if the open loop gain of the AVR loop is greater than 32.33, then the terminal voltage will be within 3% of the base voltage.

Automatic Load Frequency Control

ALFC is to control the frequency deviation by maintaining the real power balance in the system. The main functions of the ALFC are to i) to maintain the steady frequency; ii) control the tie-line flows; and iii) distribute the load among the participating generating units. The control (input) signals are the tie-line deviation ΔP_{tie} (measured from the tie- line flows), and the frequency deviation Δf (obtained by measuring the angle deviation $\Delta\delta$). These error signals Δf and ΔP_{tie} are amplified, mixed and transformed to a real power signal, which then controls the valve position. Depending on the valve position, the turbine (prime mover) changes its output power to establish the real power balance. The complete control schematic is shown in Fig2.3

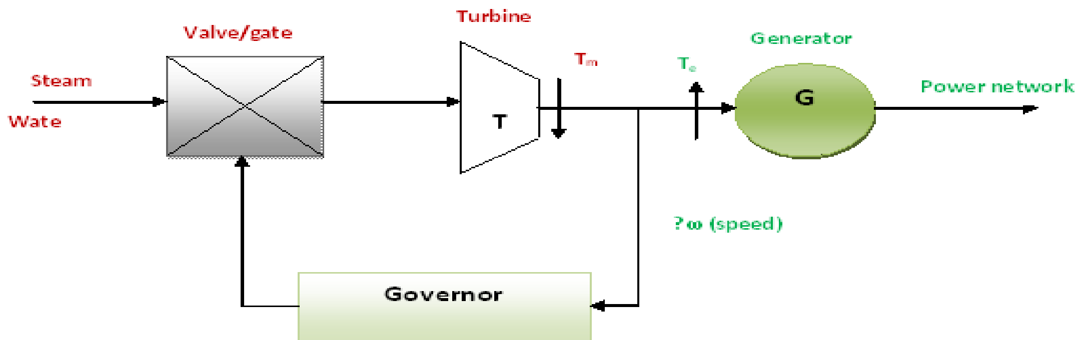


Fig2.3: The Schematic representation of ALFC system

For the analysis, the models for each of the blocks in Fig2 are required. The generator and the electrical load constitute the power system. The valve and the hydraulic amplifier represent the speed governing system. Using the swing equation, the generator can be modeled by

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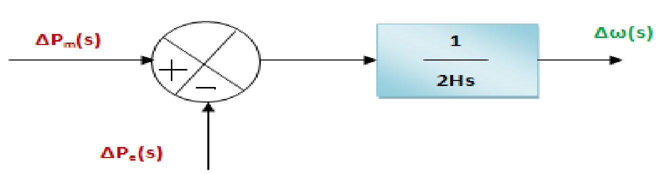


Fig2.4. The block diagram representation of the Generator

load on the system is composite consisting of a frequency independent component and a frequency dependent component. The load can be written as $\Delta P_e = \Delta P_0 + \Delta P_f$ where, ΔP_e is the change in the load; ΔP_0 is the frequency independent load component;

ΔP_f is the frequency dependent load component. $\Delta P_f = D\Delta\omega$ where, D is called frequency characteristic of the load (also called as damping constant) expressed in percent change in load for 1% change in frequency. If D=1.5%, then a 1% change in frequency causes 1.5% change in load. The combined generator and the load (constituting the power system) can then be represented as shown in Fig2.5

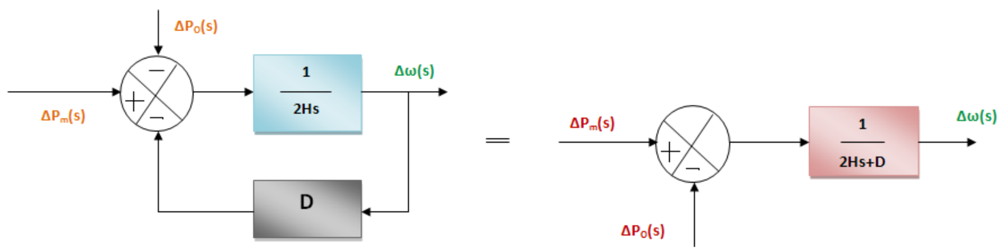
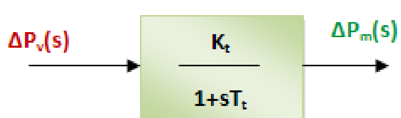


Fig2.5. The block diagram representation of the Generator and load

The turbine can be modeled as a first order lag as shown in the Fig2.6



$$G_t(s) = \frac{\Delta P_m(s)}{\Delta P_V(s)} = \frac{K_t}{1+sT_t}$$

$G_t(s)$ is the TF of the turbine; $\Delta P_V(s)$ is the change in valve output (due to action).
$\Delta P_m(s)$ is the change in the turbine output

Fig2.6. The turbine model.

The governor can similarly modeled as shown in Fig2.7. The output of the governor is by

where ΔP is the reference set power, and $\Delta\omega/R$ is the power given by governor speed characteristic. The hydraulic amplifier transforms this signal ΔP_g into valve/gate position corresponding to a power ΔP_V. Thus $\Delta P_V(s) = (K_g/(1+sT_g))\Delta P_g(s)$.

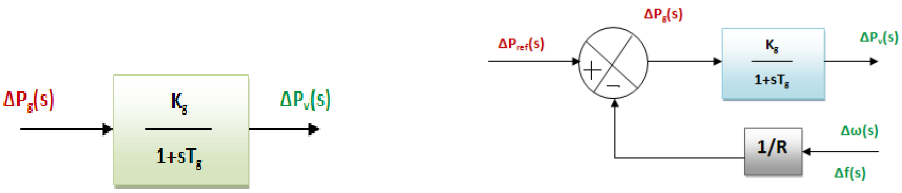


Fig2.7: The block diagram representation of the Governor

All the individual blocks can now be connected to represent the complete ALFC loop as shown in Fig2.8

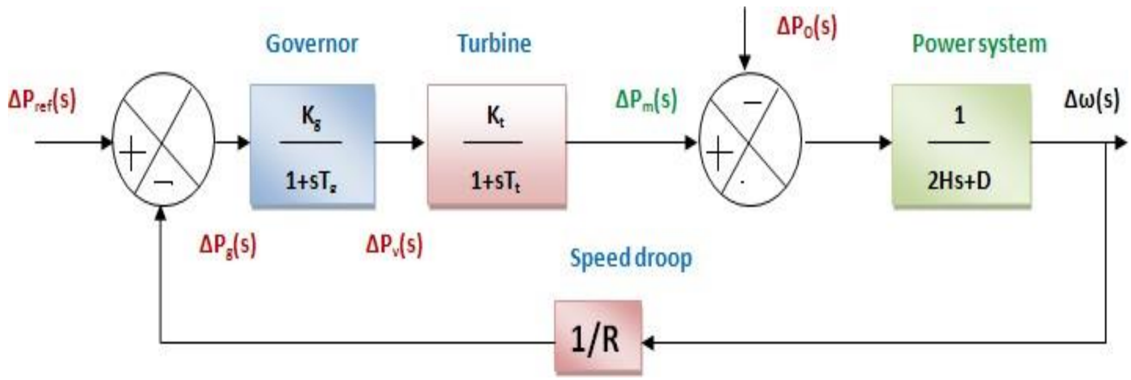


Fig2.8: The block diagram representation of the ALFC

Steady State Performance of the ALFC Loop

In the steady state, the ALFC is in 'open' state, and the output is obtained by substituting s→0 in the TF. With s→0, Gg(s) and Gt(s) become unity, then, (note that $\Delta P_m = \Delta P_T = \Delta P_G = \Delta P_e = \Delta P_D$; That is turbine output = generator/electrical output = load demand)

$$\Delta P_m = \Delta P_{ref} - (1/R)\Delta\omega \qquad \text{or} \qquad \Delta P_m = \Delta P_{ref} - (1/R)\Delta f$$

When the generator is connected to infinite bus ($\Delta f = 0$, and $\Delta V = 0$), then $\Delta P_m = \Delta P_{ref}$.

If the network is finite, for a fixed speed changer setting ($\Delta Pref = 0$), then

$$\Delta P_m = -(1/R)\Delta f \text{ or } \qquad \Delta f = -R\, \Delta P_m.$$

If the frequency dependent load is present, then

$$\Delta P_m = \Delta P_{ref} - (1/R + D)\Delta f \qquad \text{or} \qquad \Delta f = \frac{-\Delta P_m}{D + 1/R}$$

If there are more than one generator present in the system, then

$$\Delta P_{m.\ eq} = \Delta P_{ref.eq} - (D + 1/R_{eq})\Delta f$$

where, $\Delta P_{m.\ eq} = \Delta P_{m1} + \Delta P_{m2} + \Delta P_{m.3} + \ldots.$

$\Delta P_{ref.\ eq} = \Delta P_{ref1} + \Delta P_{ref2} +$

$\Delta P_{ref3} + \ldots..\ 1/R_{eq} \quad = (1/R_1 + 1/R_2$

$+ 1/R_2 + \ldots.)$

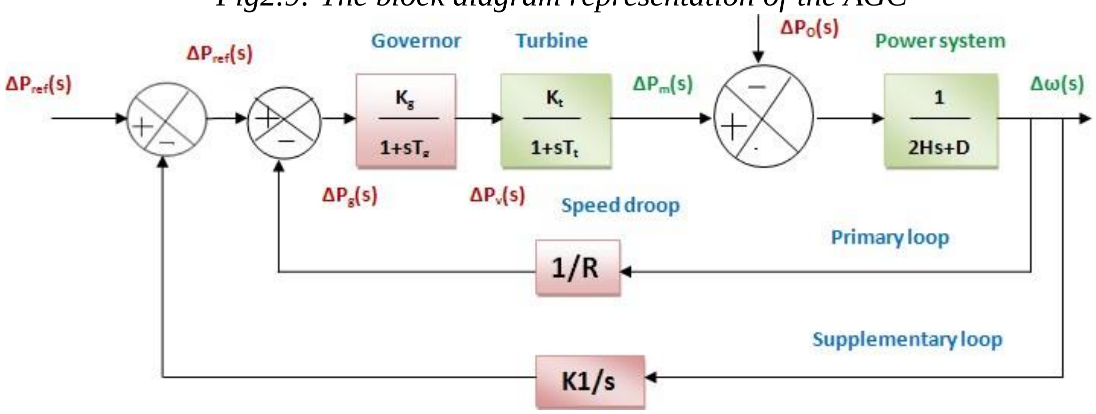
The quantity$\beta = (D + 1/R_{eq})$ is called the area frequency (bias) characteristic (response) or simply the stiffness of the system.

Concept of AGC (Supplementary ALFC Loop)

The ALFC loop shown in Fig2.8, is called the primary ALFC loop. It achieves the primary goal of real power balance by adjusting the turbine output ΔP_m to match the change in load demand ΔP_D. All the participating generating units contribute to the change in generation. But a change in load results in a steady state frequency deviation

Δf. The restoration of the frequency to the nominal value requires an additional control loop called the supplementary loop. This objective is met by using integral controller which makes the frequency deviation zero. The ALFC with the supplementary loop is generally called the AGC. The block diagram of an AGC is shown in Fig2.9. The main objectives of AGC are i) to regulate the frequency (using both primary and supplementary controls); ii) and to maintain the scheduled tie-line flows. A secondary objective of the AGC is to distribute the required change in generation among the connected generating units economically (to obtain least operating costs).

Fig2.9: The block diagram representation of the AGC



AGC in a Single Area System

In a single area system, there is no tie-line schedule to be maintained. Thus the function of the AGC is only to bring the frequency to the nominal value. This will be achieved using the supplementary loop (as shown in Fig.2.9) which uses the integral controller to change the reference power setting so as to change the speed set point. The integral controller gain K_I needs to be adjusted for satisfactory response (in terms of overshoot, settling time) of the system. Although each generator will be having a separate speed governor, all the generators in the control area are replaced by a single equivalent generator, and the ALFC for the area corresponds to this equivalent generator.

AGC in a Multi Area System

In an interconnected (multi area) system, there will be one ALFC loop for each control area (located at the ECC of that area). They are combined as shown in Fig2.10 for the interconnected system operation. For a total change in load of ΔP_D, the steady state

deviation in frequency in the two areas is given by $\Delta f = \Delta\omega_1 = \Delta\omega_2 = \frac{-\Delta P_D}{\beta_1 + \beta_2}$ where,

$\beta_1 = (D_1 + 1/R_1)$; and $\beta_2 = (D_2 + 1/R_2)$.

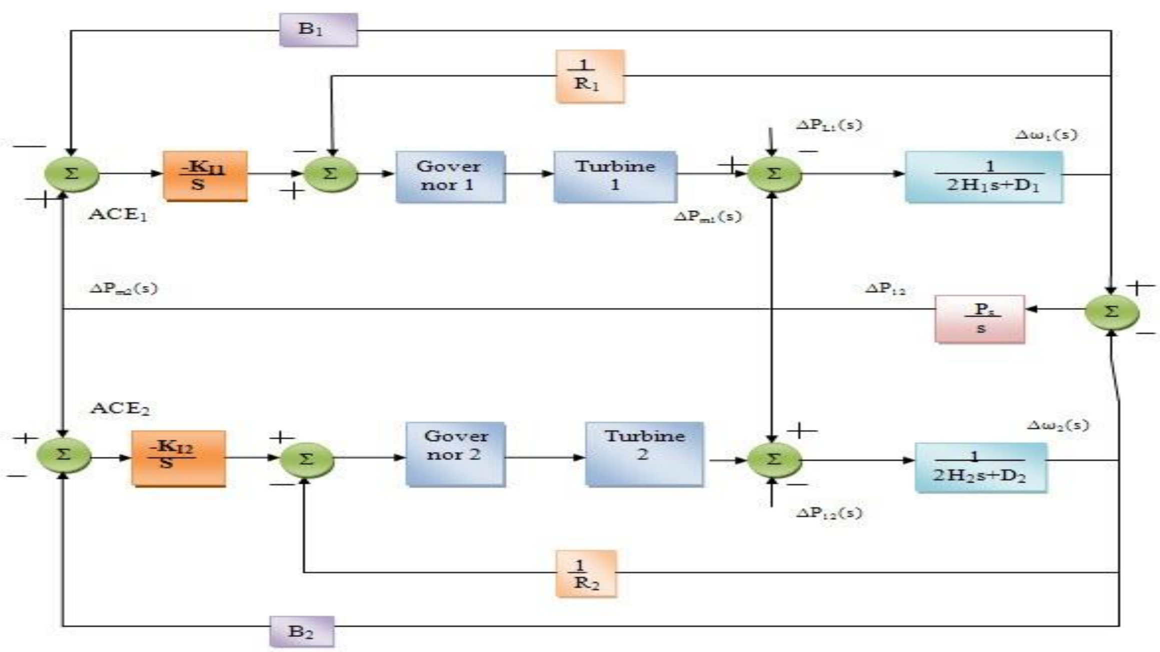


Fig.2.10. AGC for a multi-area operation.

Expression for tie-line flow in a two-area interconnected system Consider a change in load ΔP_{D1} in area1. The steady state frequency deviation Δf is the same for both the areas. That is $\Delta f = \Delta f_1 = \Delta f_2$. Thus, for area1, we have

$$\Delta P_{m1} - \Delta P_{D1} - \Delta P_{12} = D_1 \Delta f$$

where, ΔP_{12} is the tie line power flow from Area1to Area 2; and for Area 2

$$\Delta P_{m2} + \Delta P_{12} = D_2 \Delta f$$

The mechanical power depends on regulation. Hence

$$\Delta P_{m1} = -\frac{\Delta f}{R_1} \quad \text{and} \quad \Delta P_{m2} = -\frac{\Delta f}{R_2}$$

Substituting these equations, yields

$$\left(\frac{1}{R_1} + D_1\right)\Delta f = -\Delta P_{12} - \Delta P_{D1} \quad \text{and} \quad \left(\frac{1}{R_2} + D_2\right)\Delta f = \Delta P_{12}$$

Solving for Δf, we get

$$\Delta f = \frac{-\Delta P_{D1}}{(1/R_1 + D_1) + (1/R_2 + D_2)} = \frac{-\Delta P_{D1}}{\beta_1 + \beta_2}$$

and

$$\Delta P_{12} = \frac{-\Delta P_{D1}\beta_2}{\beta_1 + \beta_2}$$

where, β_1 and β_2 are the composite frequency response characteristic of Area1 and Area 2 respectively. An increase of load in area1 by ΔP_{D1} results in a frequency reduction in both areas and a tie-line flow of ΔP_{12}. A positive ΔP_{12} is indicative of flow from Area1 to Area 2 while a negative ΔP_{12} means flow from Area 2 to Area1. Similarly, for a change in Area

2 load by ΔP_{D2}, we have

$$\Delta f = \frac{-\Delta P_{D2}}{\beta_1 + \beta_2}$$

and

$$\Delta P_{12} = -\Delta P_{21} = \frac{\Delta P_{D2}\beta_1}{\beta_1 + \beta_2}$$

Frequency bias tie line control

The tie line deviation reflects the contribution of regulation characteristic of one area to another. The basic objective of supplementary control is to restore balance between each area load generation. This objective is met when the control action maintains

- Frequency at the scheduled value

- Net interchange power (tie line flow) with neighboring areas at the scheduled values

The supplementary control should ideally correct only for changes in that area. In other words, if there is a change in Area1 load, there should be supplementary control only in Area1 and not in Area 2. For this purpose the area control error (ACE) is used (Fig2.9). The ACE of the two areas are given by

For area 1: $ACE_1 = \Delta P_{12} + \beta_1 \Delta f$

For area 2: $ACE_2 = \Delta P_{21} + \beta_2 \Delta f$

Economic Allocation of Generation

An important secondary function of the AGC is to allocate generation so that each generating unit is loaded economically. That is, each generating unit is to generate that amount to meet the present demand in such a way that the operating cost is the minimum. This function is called Economic Load Dispatch (ELD).

Systems with more that two areas

The method described for the frequency bias control for two area system is applicable to multiarea system also.

Note:

The regulation constant R is negative of the slope of the Δf verses Δp_m curve of the turbine-governor control. The unit of R is Hz/MW when Δf is in Hz and Δp_m is in MW. When Δf and Δp_m are in per-unit, R is also in per-unit.

The area frequency characteristic is defined as $\beta = \{1/(D+1/R)\}$, where D is the frequency damping factor of the load. The unit of β is MW/Hz when Δf is in Hz and Δp_m is in MW. If Δf and Δp_m are in per unit, then β is also in per unit.

Examples:

Ex 1. A 500 MVA, 50 Hz, generating unit has a regulation constant R of 0.05 p.u. on its own rating. If the frequency of the system increases by 0.01 Hz in the steady state, what is the decrease in the turbine output? Assume fixed reference power setting.

Solution: In p.u. $\Delta f = 0.01/50 = 0.0002$ p.u.

With $\Delta p_{ref} = 0$, $\Delta p_m = -1/R(\Delta f) = -0.004$ p.u.

Hence, $\Delta p_m = -0.004\ S_{base} = -2$ MW.

Ex. 2. An interconnected 60 Hz power system consists of one area with three generating units rated 500, 750, and 1000 MVA respectively. The regulation constant of each unit is R= 0.05 per unit on its own rating. Each unit is initially operating at one half of its rating, when the system load suddenly increases by 200MW. Determine (i) the area frequency response characteristic on a 1000 MVA system base, (ii) the steady state frequency deviation of the area, and (iii) the increase in turbine power output.

Regulation constants on common system base are ($R_{pu\ new} = R_{pu\ old}$ ($S_{base\ new}/S_{base\ old}$):

R1 = 0.1; R2 = 0.0667; and R3 = 0.05.

Hence $\beta = (1/R1 + 1/R2 + 1/R3) = 45$ per unit.

Neglecting losses and frequency dependence of the load, the steady state frequency deviation is $\Delta f = (-1/\beta)\Delta p_m = -4.444 \times 10^{-3}$ per unit = $(-4.444 \times 10^{-3})60 = -0.2667$ Hz.

$\Delta p_{m1} = (-1/R1)(\Delta f) = 0.04444$ per unit = 44.44 MW

$\Delta p_{m2} = (-1/R2)(\Delta f) = 0.06666$ per unit = 66.66 MW

$\Delta p_{m3} = (-1/R3)(\Delta f) = 0.08888$ per unit = 88.88 MW

Ex.3. A 60 Hz, interconnected power system has two areas. Area1 has 2000 MW generation and area frequency response of 700 MW/Hz. Area 2 has 4000 MW generation and area frequency response of 1400 MW/Hz. Each area is initially generating half of its rated generation, and the tie-line deviation is zero at 60 Hz when load in Area1 is suddenly increases by 100 MW. Find the steady state frequency error and tie line error of the two areas. What is the effect of using AGC in this system?

In the steady state, $\Delta f = (-1/\beta)\ \Delta p_m = \{\Delta p_m / -(\beta 1 + \beta 2)\} = (\ -100/2100) = -\ 0.0476$ Hz.

Assuming $\Delta p_{ref} = 0$,

$$\Delta p_{m1} = -\beta 1\ \Delta f = 33.33 \text{ MW; and } \Delta p_{m2} = -\beta 2\ \Delta f = 66.67 \text{ MW.}$$

Thus in response to 100 MW change in Area1, both areas will change their generation. The increase in Area 2 generation will now flow through tie line to Area1.

Hence $\Delta p_{tie1} = -\ 66.67$ MW; and $\Delta p_{tie2} = +\ 66.67$ MW.

With AGC, the Area control error is

determined as follows. ACE 1 = $\Delta p_{tie1} + B_1$

Δf where B_1 is the frequency bias constant.

ACE 2 = Δp_{tie2} (= - Δp_{tie1}) + $B_2\ \Delta f$ where B_2 is the frequency bias constant.
The control will actuate such that in the steady state the frequency and tie line deviations are zero.
Thus till ACE1= ACE2 = 0, t

HYDROTHERMAL SCHEDULING

OPTIMAL SCHEDULING OF HYDROTHERMAL SYSTEM

- No state or country is endowed with plenty of water sources or abundant coal or nuclear fuel.
- In states, which have adequate hydro as well as thermal power generation capacities, proper co-ordination to obtain a most economical operating state is essential.
- Maximum advantage is to use hydro power so that the coal reserves can be conserved and environmental pollution can be minimized.
- However in many hydro systems, the generation of power is an adjunct to control of flood water or the regular scheduled release of water for irrigation. Recreations centers may have developed along the shores of large reservoir so that only small surface water elevation changes are possible.
- The whole or a part of the base load can be supplied by the run-off river hydro plants, and the peak orthe remaining load is then met by a proper mix of reservoir type hydro plants and thermal plants. Determination of this by a proper mix is the determination of the most economical operating state of a hydro-thermal system. The hydro-thermal coordination is classified into long term co-ordination and short term coordination.

The previous sections have dealt with the problem of optimal scheduling of a power system with thermal plants only. Optimal operating policy in this case can be completely determined at any instant without reference to operation at other times. This, indeed, is the static optimization problem. Operation of a system having both hydro and thermal plants is, however, far more complex as hydro plants have negligible operating cost, but are required to operate under constraints of water available for hydro generation in a given period of time. The problem thus belongs to the realm of dynamic optimization. The problem of minimizing the operating cost of a hydrothermal system can be viewed as one of minimizing the fuel cost of thermal plants under the constraint of water availability (storage and inflow) for hydro generation over a given period of operation.

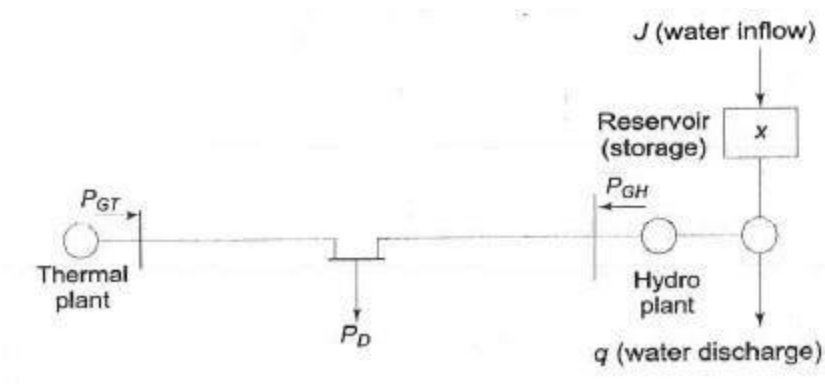


Fig. 2.1 Fundamental hydrothermal system

For the sake of simplicity and understanding, the problem formulation and solution technique are illustrated through a simplified hydrothermal system of Fig. 2.1. This system consists of one hydro and one thermal plant supplying power to a centralized load and is referred to as a fundamental system. Optimization will be carried out with real power generation as control variable, with transmission loss accounted for by the loss formula.

Mathematical Formulation

For a certain period of operation T (one year, one month or one day, depending upon the requirement), it is assumed that (i) storage of hydro reservoir at the beginning and the end of the period are specified, and (ii) water inflow to reservoir (after accounting for irrigation use) and load demand on the system are known as functions of time with complete certainty (deterministic case). The problem is to determine q(t), the water discharge (rate) so as to minimize the cost of thermal generation.

$$C_T = \int_0^T C'\left(P_{GT}(t)\right) dt \qquad (3.1)$$

under the following constraints:

(i) Meeting the load demand

$$P_{GT}(t) + P_{GH}(t) - P_L(t) - P_D(t) = 0; \; t \in [0,T] \qquad (3.2)$$

This is called the power balance equation.

(ii) Water availability

$$X'(T) - X'(0) - \int_0^T J(t)dt + \int_0^T q(t)dt = 0 \quad (3.3)$$

where J(t) is the water inflow (rate), X'(t) water storage, and X'(0) , X' (T) are specified water storages at the beginning and at the end of the optimization interval.

(iii) The hydro generation $P_{GH}(t)$ is a function of hydro discharge and water storage (or head), i.e.

$$P_{GH}(t) = f(X'(t),q(t)) \quad (3.4)$$

The problem can be handled conveniently by discretization. The optimization interval T is subdivided into M subintervals each of time length ΔT. Over each subinterval it is assumed that all the variables remain fixed in value. The problem is now posed as

$$min\ \Delta T \sum_{m=1}^{M} C'(P^m_{GT}) = min \sum_{m=1}^{M} C(P^m_{GT}) \quad (3.5)$$

under the following constraints:

(i) Power balance equation

$$P^m_{GT} + P^m_{GH} - P^m_L - P^m_D = 0 \quad (3.6)$$

where

P^m_{GT} = thermal generation in the mth interval

P^m_{GH} = hydro generation in the mth interval

P^m_L =transmission loss in the mth interval

$$= B_{TT}(P^m_{GT})^2 + 2B_{TH}P^m_{GH} + B_{HH}(P^m_{GH})^2$$

P^m_D = load demand in the mth interval

(ii) Water continuity equation

$X'^m - X'^{(m-1)} - J^m\ \Delta T + q^m\ \Delta T = 0$

where

X'^m = water storage at the end of the mth interval

J^m = water inflow (rate) in the mth interval

q^m = water discharge (rate) in the mth interval

The above equation can be written as

$$X^m - X^{(m-1)} - J^m + q^m = 0;\ m = 1,2,\ \ldots,M \qquad (3.7)$$

where $X^m = X'^m/\Delta T$ = storage in discharge units.

In Eqs. (3.7), X^o and X^M are the specified storages at the beginning and end of the optimization interval.

(iii) Hydro generation in any subinterval can be expressed as

$$P_{GH}^m = h_o\ 1 + 0.5e(X^m + X^{m-1})\quad q^m - \rho \qquad (3.8)$$

where

$h_o = 9.81 \times 10^{-3}\ h'_o$

h_o = basic water head (head corresponding to dead storage)

e = water head correction factor to account for head variation with storage

ρ = non-effective discharge (water discharge needed to run hydro generator at no load).

In the above problem formulation, it is convenient to choose water discharges in all subintervals except one as independent variables, while hydro generations, thermal generations and water storages in all subintervals are treated as dependent variables. The fact, that water discharge in one of the subintervals is a dependent variable, is shown below:

Adding Eq. (3.7) for m = l, 2, ...,M leads to the following equation, known as water availability equation

$$X^M - X^0 - \sum_m J^m + \sum_m q^m = 0 \qquad (3.9)$$

Because of this equation, only (M - l) qs can be specified independently and the remaining one can then be determined from this equation and is, therefore, a dependent variable. For convenience, q^1 is chosen as a dependent variable, for which we can write

$$q^1 = X^0 - X^M + \sum_m J^m - \sum_{m=2}^{M} q^m \qquad (3.10)$$

Solution Technique

The problem is solved here using non-linear programming technique in conjunction with the first order gradient method. The Lagrangian $\mathcal{L}$ is formulated by augmenting the cost function of Eq. (3.5) with equality constraints of Eqs. (3.6)-(3.8) through Lagrange multipliers (dual variables) λ_1^m, λ_2^m and λ_3^m. Thus,

$$\mathcal{L} = \sum_m \Big[C(P^m_{GT}) - \lambda_1^m \left(P^m_{GT} + P^m_{GH} - P^m_L - P^m_D\right) + \lambda_2^m \left(X^m - X^{m-1} - J^m + q^m\right) + \lambda_3^m \left\{P^m_{GH} - h_o\left(1 + 0.5e(X^m + X^{m-1})\right)(q^m - \rho)\right\}\Big] \qquad (3.11)$$

The dual variables are obtained by equating to zero the partial derivatives of the Lagrangian with respect to the dependent variables yielding the following equations

$$\frac{\partial \mathcal{L}}{\partial P^m_{GT}} = \frac{dC(P^m_{GT})}{dP^m_{GT}} - \lambda_1^m \left(1 - \frac{\partial P^m_L}{\partial P^m_{GT}}\right) = 0 \qquad (3.12)$$

$$\frac{\partial \mathcal{L}}{\partial P^m_{GH}} = \lambda_3^m - \lambda_1^m \left(1 - \frac{\partial P^m_L}{\partial P^m_{GH}}\right) = 0 \qquad (3.13)$$

$$\left.\frac{\partial \mathcal{L}}{\partial X^m}\right|_{m \neq M, \neq 0} = \lambda_2^m - \lambda_2^{m+1} - \lambda_3^m (0.5 h_o e)(q^m - \rho) - \lambda_3^{m+1} (0.5 h_o e)(q^{m+1} - \rho) = 0 \qquad (3.14)$$

(3.15)

The dual variables for any subinterval may be obtained as follows:

(i) Obtain λ_1^m from Eq. (3.12).

(ii) Obtain λ_3^m from Eq. (3.13).

(iii) Obtain λ_2^1 from Eq. (3.15) and other values of λ_2^m $(m \neq 1)$ from Eq. (3.14).

The gradient vector is given by the partial derivatives of the Lagrangian with respect to the independent variables. Thus

$$\left.\frac{\partial \mathcal{L}}{\partial q_m}\right|_{m \neq 1} = \lambda_2^m - \lambda_3^m h_o \left[1 + 0.5\, e \left(2X^{m-1} + J^m - 2q^m + \rho\right)\right] \qquad (3.16)$$

For optimality the gradient vector should be zero if there are no inequality constraints on the control variables.